**Homework 11 Do NOT Turn In**

1. Roebuck is a manufacturer of sweatshirts and has an exclusive contract to sell sweatshirts to the CU community (i.e. Roebuck is a monopolist). The total cost curve of manufacturing white sweatshirts is **TC(Q) = 40Q.** The CU community is composed of students and employees. The demand curve of students for these sweatshirts is **Q1 = 88 – P1**. Q1 is the number of sweatshirts demanded by students and P1 is the price paid by the students. The demand curve of employees is **Q2 = 120 – P2**. Q2 is the quantity demanded by employees and P2 is the price paid by employees.

1. Suppose that Roebuck must charge students and employees the same price. What price will Roebuck charge? How many sweatshirts will Roebuck sell to students? to employees?
2. What is the elasticity of demand for each type of customer at the common price? Looking at these elasticities can you tell which type of customer (student or employee) that Roebuck would like to charge a higher price if it was able to set two different prices? Briefly explain your answer.
3. Suppose that Roebuck may be able to charge different prices. In order to price discriminate Roebuck must be able to tell who is a student and who is an employee. Suppose that Roebuck does not have that information. However the firm knows that students are indifferent between a white sweatshirt and a fuchsia sweatshirt (i.e. students will always purchase the cheaper of the two sweatshirts), but that employees will only purchase white sweatshirts (i.e. regardless of the prices employees will never purchase a fuchsia sweatshirt). The cost of dying the white sweatshirt fuchsia is $6 per sweatshirt. What price will the firm set for white sweatshirts? What price will it set for the fuchsia sweatshirts? How many of each sweatshirt will it sell?

2. In the following question you are going to analyze how the firm’s profit maximizing level of output adjusts to changes in the firms revenues. In all of the parts below you will assume that the firms cost curve is **C(Q)= cQ**.

An *ad valorem* tax is a percentage tax on the price that a firm charges. Specifically if the firm is charging P(Q) then after the imposition of an *ad valorem* tax the firm would receive only (1-t)P(Q) where t is the tax rate.

1. Write the firm’s profit function as a function of Q and t. Write the first order conditions that define the profit maximizing level of output QM for the firm.

For each tax rate t a profit maximizing firm must choose a QM such that the first order conditions above hold. Thus the first order conditions define an **implicit** relationship between t and QM.

1. Using your first order conditions find the derivative  of the implicitly defined relationship between QM and t.
2. Suppose that P(Q) = a – bQ. Show that you can write the derivative from part (b) as 
3. Last week you considered the effect of an *ad valorem* tax on a Mr. Wagyu had a constant marginal cost of 0. Use the formula from part c to calculate the effect of an ad valorem tax on Mr Wagyu’s profit maximizing level of output. What is the effect on the price that Mr Wagyu charged?

The *ad valorem* tax above is an example of a parameter that affects the firm’s revenue function. In particular we can write the revenue function for that example R(Q,t) = (1-t)P(Q)Q. In general there may be any number of parameters that affect the firms revenues and we can write the revenue function as R(Q,s) where s is some parameter.

1. Write the firm’s profit function as a function of Q and s. Write the first order conditions that define the profit maximizing level of output QM for the firm.

For each s a profit maximizing firm must choose a QM such that the first order conditions above hold. Thus the first order conditions define an implicit relationship between s and QM.

1. Using your first order conditions find the derivative  of the implicitly defined relationship between QM and s.

3. Suppose that there are three identical firms in the vitamin industry. The cost curve of each firm is **C(Q) = .5Q2.** Quantity (Q) is measured in tons per day. Demand per day for vitamins is **QD = 720 – 2P** whereprice (P) is measured in dollars per ton.

1. If the three firms are price takers then what are the supply curves of each firm? What is the industry supply curve?
2. If the three firms behave as price takers then what will be the price and quantity that clear the market?
3. If the three firms formed a cartel then what would be the marginal cost curve of the cartel?
4. If the cartel is a monopolist then what price will the cartel charge for vitamins? What quantity will it sell? Illustrate the demand, marginal revenue and marginal cost curves in a diagram on the next page. You do not need to illustrate the individual marginal cost curves in this diagram. It is sufficient to illustrate only the marginal cost curve of the cartel.

Assume that the firms in the cartel divide the production and the profits equally among the 3 members.

1. **W**hat are the profits of an individual firm in the cartel?
2. Suppose that firm 1 “cheats” on the cartel by producing 2 more tons than its allotted equal share. If neither firm 2 nor firm 3 cheats (so they produce the same number of tons as they did in part (e)) and firm 1 produces 2 more tons then what will be the new price in the market? Show that firm 1 will earn more by cheating than it did in part (e).
3. Briefly explain why it is that firm 1 could increase its individual profits by increasing production from the cartel solution but that cartel itself could not increase profits by increasing production. To receive full credit you must refer to infra-marginal losses.

4. The Jolly Giant is the unique supplier of premium canned peas to Rogerson’s Supermarkets. The daily demand at Rogerson’s for Jolly Giant’s peas is given by the inverse demand function **P(Q) = 258 - 2Q** where Q is measured in cans of peas and P in cents per can. The Jolly Giant’s daily marginal cost of canned pea production is given by **MC(Q) = 2Q + 6** where cost is also measured in cents per can.

1. Given the demand and cost information how many cans of peas will the Jolly Giant choose to supply each day? What will be the price in the market? What will be the marginal revenue at this level of sales? Illustrate your answer. Be sure to label the quantity supplied, the price charged and the marginal revenue.

Rogerson’s stocks “generic” canned peas in addition to the Jolly Giant’s premium canned peas. The generic peas appeal to the customer that always chooses the peas that are priced the lowest. In fact the demand for generic peas is perfectly elastic at a price of $1.10 (110 cents). The cost to Jolly Giant of producing generic peas is the same as the cost of producing premium peas.

1. If the Jolly Giant believes that it can sell as many cans of generic peas as it would like in Rogerson’s at the price of $1.1 per can then what will the marginal revenue from sales of generic peas be? If the inverse demand curve for Jolly Giants premium can peas is unchanged and the Jolly Giant can sell generic peas then without calculating the new quantities will the Jolly Giant choose to supply the same quantity of premium peas as it did previously? Briefly explain your answer.
2. Given the demand for generic peas and the inverse demand curve for premium peas how many cans of each type of peas would the Jolly Giant like to supply? What price will it charge for premium peas?
3. Illustrate the prices charged and the quantities supplied for each type of pea in your diagram for part (b).

5. A firm operates two factories. The cost function at the first plant is **C1(Q1) = (Q1)2 + 8Q1 + 5** and the cost function at the second factory is **C2(Q2) = 2(Q2)2 + 2Q2 + 10**. Find and graph the marginal cost curve for the firm. What is the firm’s supply curve?

6. The *Zeroz Company* is the leading manufacturer of photocopying machines. In addition to making the machines the company also makes all of the replacement parts including the toner cartridges. The demand per month for toner cartridges is given by **QD = 11000 – 10P**. Quantity is measured in cases and price is measured in dollars per case. The marginal cost to *Zeroz* of manufacturing a case of toner cartridges is constant at $60 per case.

1. If *Zeroz* is the only manufacturer of toner cartridges for its machines then how many cartridges will *Zeroz* choose to sell? What will be the price of a case of toner cartridges? What will be the producer surplus that *Zeroz* earns on sales of toner cartridges?

The *Office Supply Company* is a small manufacturing firm that produces toner cartridges for many different office machines. Because of the high price being charged by *Zeroz* the *Office Supply Company* decides to enter the market for *Zeroz* toner cartridges. The marginal cost to the *Office Supply Co*. of producing toner cartridges for the *Zeroz* machines is given by **MC(QOS) = QOS**. QOS is also measured in cases and it represents the number of cartridges produced by the *Office Supply Co*.

1. Assume that the *Office Supply Co*. is a price taker in the market for *Zeroz* cartridges, that is the *Office Supply Co*. takes the price charged by *Zeroz* as fixed, then how many cartridges will the *Office Supply Co*. want to sell at the price that you found in part (a)? If the *Office Supply Co*. supplies this amount while the *Zeroz Co*. continues to supply the amount from part (a) what will happen to the price in the market? What will the new price be?

As you see in your answer to (b) after a firm enters the market the price that *Zeroz* was charging in order to maximize its profits will no longer prevail. Hence the quantity that *Zeroz* chose in part (a) may no longer be profit maximizing. You will use the following steps to analyze the situation between *Zeroz* and the *Office Supply Co*.

1. *Zeroz* announces a price first and guarantees that it will meet demand at that price.
2. After observing the price set by *Zeroz*, the *Office Supply Co*. will choose the quantity that it would like to sell at that price (it is a price taker at the price set by *Zeroz*).
3. Given the price announced by *Zeroz* at the beginning market demand is determined. The sales of the *Office Supply Co* are equal to those that it chose. The sales of *Zeroz* are equal to the difference between market demand and the sales of the *Office Supply Co*. Profits are determined for each firm based on the number of sales, the price in the market and costs. You may assume that neither firm has fixed costs of production.
4. Given the supply function of the *Office Supply Co*., find the residual demand curve facing *Zeroz*.
5. Using the residual demand curve, find *Zeroz* profit maximizing level of sales. What is the price that *Zeroz* announces? What is the total quantity sold in the market?